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GALACTIC STARS APPLIED TO TESTS OF THE CRITERION FOR CONVECTION AND SEMICONVECTION IN AN INHOMOGENEOUS STAR

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ABSTRACT

The question of the effective criterion for convection and semiconvection to use in the inhomogeneous layers of models of massive stars is addressed here in terms of a choice between the Schwarzschild (temperature-gradient) criterion and the Ledoux (density-gradient) criterion. These two extremes give rise to and suppress, respectively, a fully convective zone (FCZ) in the layers immediately above the hydrogen-burning shell, if the star's mass is neither too low nor too high. Specifically, the existence of a large FCZ delays or prevents the star during core helium burning from becoming a red supergiant. The applicable range of initial stellar masses for a solar metallicity is $\sim$13–30 $M_\odot$. Nine different tests for the effective criterion for convection and semiconvection in Galactic supergiants, as well as a test using SN 1987A of the Large Magellanic Cloud, are performed, or reexamined, by using both old and new theoretical data, while three of the Galactic tests are repeated for association supergiants separately from cluster supergiants. Although eight of the 13 tests yield inconclusive results, five do support the Ledoux criterion, and three do so strongly.

Subject headings: convection — stars: early type — stars: individual (SN 1987A) — stars: interiors — supergiants

1. INTRODUCTION

Convective instability breaks out in a chemically homogeneous region of a star if the Schwarzschild (1906) criterion is violated—specifically, if the radiative temperature gradient becomes steeper than the adiabatic temperature gradient, $\nabla_{\rm rad} > \nabla_{\rm ad}$, where $\nabla = d\ln T/d\ln P$. But what criterion should apply in an inhomogeneous region containing a gradient of mean molecular weight μ? Ledoux (1947) argued that the original form of the Schwarzschild criterion, which implicitly compared radiative and adiabatic density gradients, should apply, and that, in this case, an outward decrease of μ would exert a stabilizing influence. Several linearized stability analyses of stellar models have been used to find support for both the Schwarzschild criterion (Schwarzschild & Härm 1958; Kato 1966; Gabriel 1970; Shibahashi & Osaki 1976; Langer, Sugimoto, & Fricke 1983) and the Ledoux criterion (Sakashita & Hayashi 1959, 1961; Gabriel 1969; Auré 1971; Mimura & Suda 1971; Stevenson 1977). Laboratory and oceanographic experiments with salinity gradients in water—sometimes also used as evidence—may be (Spiegel 1969; Spruit 1992) or may not be (Gabriel 1970; Stevenson 1977) applicable to the astrophysical situation. Although a recent theoretical study suggests that the Ledoux criterion is irrelevant for convection, even in principle (Grossman, Narayan, & Arnett 1993), the laboratory evidence has indicated otherwise (Spiegel 1969, 1972).

Supposing the correct criterion were known, what would be the final state of convective motions set up in the inhomogeneous zone? It was originally assumed that the criterion for convective instability would govern also the state of convective neutrality (semiconvection) that would eventually be attained in the unstable layers containing the μ gradient (Ledoux 1947; Schwarzschild & Härm 1958; Sakashita & Hayashi 1959, 1961). This assumption, however, cannot hold in all cases, because circumstances were later found in which unburned hydrogen outside the convective core is unable to be formally conserved by an assumed slow semiconvective mixing process, and therefore a rapidly mixed, fully convective zone (FCZ) has to develop at the bottom of the semiconvective layers (Stothers 1966; Kippenhahn 1969; see also the less rigorous models of Iben 1966). The physical necessity for this FCZ, as well as some related mathematical considerations (Auré 1971; Stevenson 1977; Langer et al. 1983; Langer, El Eid, & Fricke 1985; Umezu & Nakakita 1988; Spruit 1992; Grossman et al. 1993), has opened up to question any simple link between the criterion for convective instability and the final state of the mixed layers.

In an actual application of any convective/semiconvective prescription to stellar evolution, what finally matters most is the shape of the convectively altered μ profile (Stothers & Chin 1968; Chiosi & Summa 1970). If only a semiconvective zone develops, the change of the μ profile is relatively slight, and therefore its effect on stellar evolution is unimportant, as for example during the main-sequence phase (Stothers 1970). The subsequent generation of a large FCZ, however, can lead to substantial differences in the star's structure, delaying or preventing its evolution into a red supergiant. This development occurs more easily for the less stringent Schwarzschild criterion and also for higher stellar masses (owing to the destabilizing influence of radiation pressure). When full convection breaks out, it does so in the layers immediately above the hydrogen-burning shell, shortly after the star has left the main sequence. Two extreme possible cases can be potentially differentiated by using suitable observations of sufficiently massive stars: (1) a weak-convection case based, for simplicity, on the Ledoux criterion, which leads to an unmixed or a semiconvectively mixed state of the stratified layers in and above the hydrogen-burning shell; and (2) a strong-convection case based on the Schwarzschild criterion, as a result of which a large FCZ forms in this critical developing region. The term "criterion for convection," therefore, will here refer to those two cases. In the present paper, we shall not consider any stellar masses so high that use of the Ledoux criterion could also potentially lead to a large FCZ (Stothers 1966; Chiosi & Summa 1970).

A recent observational test to decide between the two cases has been carried out for the evolved stars in NGC 330, a metal-poor cluster in the Small Magellanic Cloud (SMC) (Stothers & Chin 1992a, b). The Ledoux criterion, in effect, appears to be strongly favored by the large number of observed red supergiants—a conclusion supported by two other recent studies of this cluster (Caloi et al. 1993; Brocato & Castellani 1993). On the other hand, the evolutionary tracks of Schaerer et al. (1993) include moderate convective core overshooting, which tends to suppress convective instability in the μ-gradient region; as a result, their tracks based on the Schwarzschild criterion look very much like our tracks based on the Ledoux criterion at the relevant stellar masses of $\sim 12\ M_\odot$. It could of course be argued that their choice of the overshoot parameter is at the upper limit of what seems to us to be observationally plausible according to independent evidence derived from B- and A-type main-sequence stars (Stothers & Chin 1991, 1993), the Sun (Antia & Chitre 1993; Monteiro, Christensen-Dalsgaard, & Thompson 1994; Basu, Antia, & Narasimha 1994), and Population I evolved supergiants (Stothers 1991; Stothers & Chin 1991; Evans, Arellano Ferro, & Udalska 1992). But unless this upper limit on overshooting is significantly reduced with more confidence, the criterion for convection cannot be safely based on the stars in NGC 330.

Earlier observational tests for the criterion for convection employed Galactic and Large Magellanic Cloud (LMC) supergiants, for which the results were very mixed (§ 5). By using these supergiants, however, higher stellar masses can be considered than in NGC 330. For such stars, the predicted divergence of the evolutionary tracks should be less sensitive to convective core overshooting. The various past problems encountered in using Galactic and LMC supergiants will be rediscussed in the present paper with the help of new models (§§ 2, 3) and better observational data, leading to some improved tests (§§ 4, 5). It turns out that the effective criterion to use does appear to be the Ledoux criterion.

2. INPUT PHYSICS

Physical input parameters used for the new stellar models are essentially the same as those adopted in our other recent work. Specifically, we take $X_e = 0.700$ as the initial hydrogen abundance by mass and $Z_e = 0.02$ or 0.03 as the initial metals abundance.

New high-temperature opacities in tabular form have been provided by the Livermore OPAL group (Iglesias, Rogers, & Wilson 1992). The moderate differences between the older Rogers & Iglesias (1992) opacities and these newer opacities arise from the inclusion of the spin-orbit interaction in the iron calculations and a reduction of the adopted abundance of iron, but, surprisingly, they exert a negligible effect on the structure of the stellar models (Stothers & Chin 1993). Cox-Stewart opacities without any molecular contributions are used to cover the low-temperature region below 6×10^3 K and the high-temperature region above 10^8 K.

Molecular opacities were included in a repeat calculation of the evolutionary sequences for the Ledoux criterion with $Z_e = 0.02$ by using a simple formula fit (Stothers & Chin 1993) that roughly matches the low-temperature opacities calculated by Alexander, Johnson, & Rypma (1983), Alexander, Augason, & Johnson (1989), and Sharp (1992). Since these evolutionary sequences turned out to be insignificantly different from the analogous nonmolecular sequences (see the comparison in Stothers & Chin 1993), we present results only for the latter, as the molecular opacities are still rather uncertain.

Nuclear reaction rates are unchanged from before. As in our other recent work on Galactic stars, we adopt for the $^{12}C(\alpha, \gamma)^{16}O$ reaction the "large" rate, which is 4 times the older rate given by Fowler, Caughlan, & Zimmerman (1975). We repeated our calculations by using the "small" rate for the evolutionary sequences with $Z_e = 0.02$, since current rates fall between these two extremes (Kettner et al. 1982; Redder et al. 1987; Zhao et al. 1993; Buchmann et al. 1993).

Convection and semiconvection are treated here in the two ways proposed in § 1. Even if semiconvection actually occurs as a series of fully convective, stepped layers separated by semiconvective diffusive layers (Spiegel 1972; Spruit 1992), the resulting chemical profile can be adequately approximated by a smooth composition gradient satisfying the neutrality condition $\nabla_{rad} = \nabla_{ad}$, without much alteration of the evolutionary track on the H-R diagram (Schlesinger 1975; Langer et al. 1985; Langer 1991b). The semiconvective composition gradient under the Schwarzschild criterion will therefore be approximated in this traditional way. The more restricted mixing by semiconvection under the Ledoux criterion, however, will be replaced by the approximation of no mixing at all (Stothers & Chin 1973, 1975); to be certain, we have recalculated the evolutionary sequences for $Z_e = 0.02$ by including Ledoux-type semiconvective mixing, and have obtained essentially identical results to the cases of no mixing, because the semiconvective zone is relatively small and the μ gradient is hardly altered at all. Finally, we have also computed a set of evolutionary sequences including convective core overshooting with a fixed ratio of overshoot distance to local pressure scale height, $d/H_P = 0.2$. The ratio of convective mixing length to local pressure scale height in the outer convective envelope, α_P, is likewise assigned to be a constant along an evolutionary track; values in the range 1.4–2.0 are found to achieve rough agreement between the predicted and observed effective temperatures of the Sun as well as of normal red supergiants.

Initial stellar masses, chosen to be 10, 15, and 20 $M_\odot$, cover the very small mass range needed. Stellar wind mass loss has been included by utilizing the parameterization of observed mass-loss rates published by Nieuwenhuijzen & de Jager (1990). In addition, we have recomputed the evolutionary sequences for $Z_e = 0.02$ without any mass loss at all. Because the rates of mass loss from red supergiants rise steeply with stellar mass and are uncertain by up to a factor of 10 either way (Dupree 1986; Jura & Kleinmann 1990), we have not considered initial stellar masses greater than 20 $M_\odot$.

Finally, rotation has been ignored, following Endal & Sofia (1976) and Howard (1993), who included it in evolutionary sequences without and with large FCZs, respectively. At most, its inclusion would be expected to add some scatter to our model results, but not to change our conclusions qualitatively.

3. NEW EVOLUTIONARY SEQUENCES

As soon as a massive star has exhausted its central hydrogen, three important factors come into play in determining its future evolution. One factor is the amount of mass already lost on the main sequence via the stellar wind. However, this factor is relatively minor for the 10–20 $M_\odot$ initial masses considered here. A second factor is the very steep gradient of the radiative opacity near the stellar surface, which acts to expand the star's radius. The third factor is the development (or nondevelopment) of a large FCZ, which, once having formed, rapidly homogenizes the highly unstable layers with a μ gradient immediately above the hydrogen-burning shell, and so acts to slow down the envelope expansion. Figure 1 illustrates the hydrogen profiles for the cases with and without an FCZ.

In the case of a normal Population I metallicity, the atomic opacities near the stellar surface turn out to be very high compared to the electron-scattering values dominant deeper in the envelope. The resulting opacity gradient through the upper envelope is sufficiently steep to force the stellar models at 10 $M_\odot$ to attain red supergiant dimensions before the onset of core helium depletion. A blue loop on the H-R diagram subsequently forms as soon as the hydrogen-burning shell has consumed its way outward either to the previously established base of the small FCZ (in the case of the Schwarzschild criterion) or to the previously established base of the outer convection zone (in the case of the Ledoux criterion).

A higher stellar masses, however, the evolutionary history depends more crucially on which criterion for convection is adopted. With the Ledoux criterion, no FCZ develops at any mass considered, and so all the evolutionary sequences resemble those for 10 $M_\odot$. At 20 $M_\odot$, however, the blue loop becomes completely suppressed by the fairly large amount of red supergiant mass loss (15% of the initial mass) if the Nieuwenhuijzen & Jager (1990) parameterized rate is used (see also Bressan et al. 1993). Since the true observational uncertainty of the average mass-loss rate for massive red supergiants could well be an order of magnitude either way (§ 2), we have reduced the parameterized rate by a factor of 5 for log $T_e < 3.65$ and log $(L/L_\odot) > 4.8$. Our model results are shown in Figures 2 and 3. In light of the fact that properties of blue loops are usually found to be quite robust once a blue loop develops, the results shown in the two figures probably have only a small uncertainty (cf. Chin & Stothers 1991). We have verified this point in the possibly questionable case of 20 $M_\odot$ by calculating two additional evolutionary sequences, one using a mass-loss rate reduction factor of 10 and the other using an exaggerated convective mixing-length parameter of $\alpha_P = 3$; the resulting blue loops are essentially the same as those shown in the figures.

When the Schwarzschild criterion for convection is adopted, the stellar models with initial masses above 10 $M_\odot$ develop a large FCZ soon after central hydrogen exhaustion. The resulting large-scale homogenization of the inner envelope eventually halts the radius expansion while the star is still a

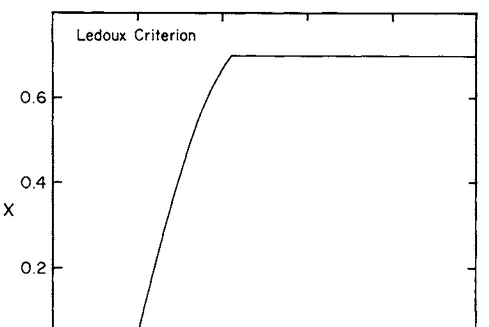

FIG. 1.—Hydrogen profiles in models of stars of initially 15 $M_\odot$ based on the Ledoux criterion and Schwarzschild criterion. The stage of evolution shown occurs not long after central hydrogen exhaustion, when convective instability in the μ-gradient region reaches its greatest extent. Stellar mass fraction is denoted q.

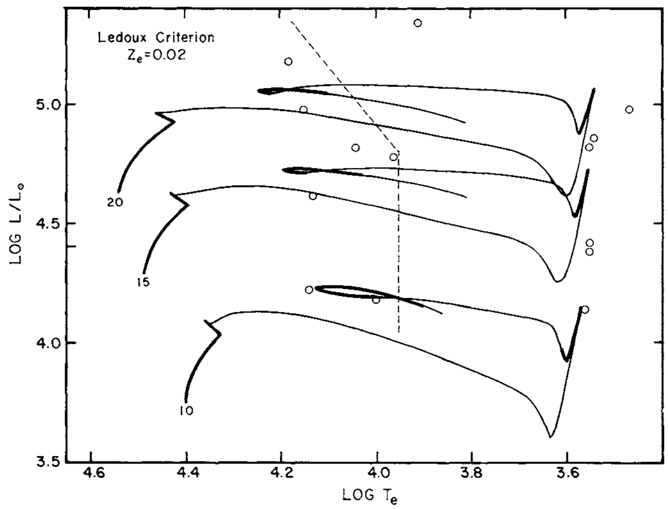

FIG. 2.—Theoretical H-R diagram showing evolutionary tracks based on the Ledoux criterion for convection. The initial metals abundance is $Z_e = 0.02$. Initial stellar masses are indicated in solar units. Tracks run from the zero-age main sequence to the end of core helium burning. Heavy segments of the lines denote slow phases. A minor adjustment of the lowest effective temperatures was made for visual purposes. Circles represent observed bright stars to the right of the main sequence in young Galactic clusters. The dashed line refers to the yellow edge of the observed distribution of blue supergiants in Galactic associations.

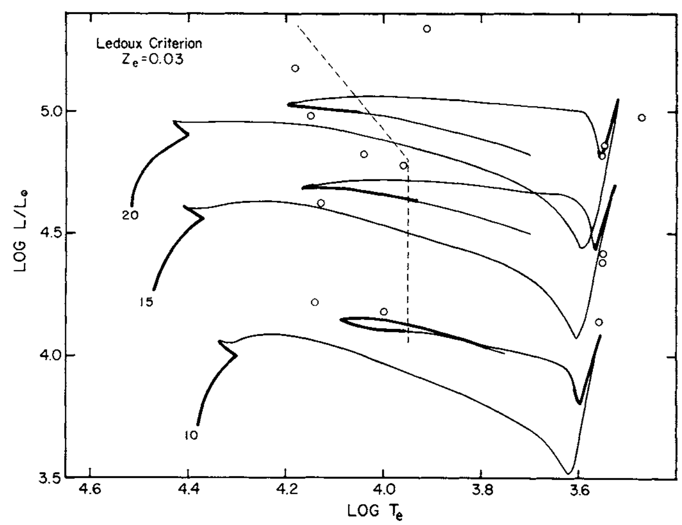

FIG. 3.—Theoretical H-R diagram showing evolutionary tracks based on the Ledoux criterion for convection. The initial metals abundance is $Z_e = 0.03$. Same notation as for Fig. 2.

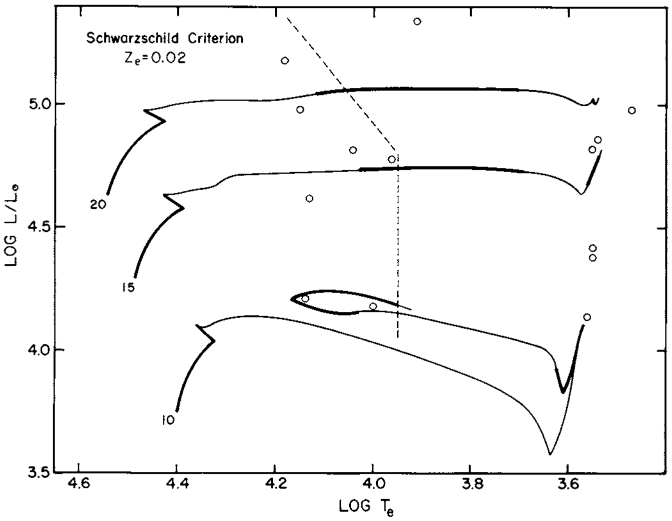

FIG. 4.—Theoretical H-R diagram showing evolutionary tracks based on the Schwarzschild criterion for convection. The initial metals abundance is $Z_e = 0.02$. Same notation as for Fig. 2.

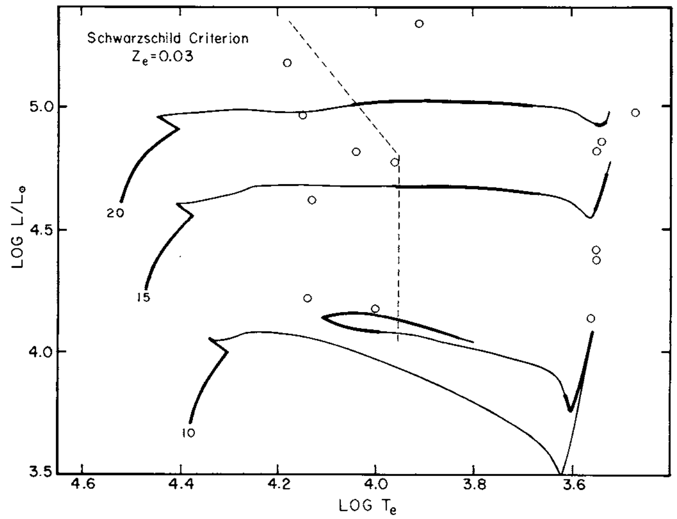

FIG. 5.—Theoretical H-R diagram showing evolutionary tracks based on the Schwarzschild criterion for convection. The initial metals abundance is $Z_e = 0.03$. Same notation as for Fig. 2.

blue supergiant. Then the star evolves very slowly through the region of blue supergiants. After a rapid sprint across the narrow yellow supergiant gap, the star finishes core helium burning as a red supergiant (Figs. 4 and 5).

Table 1 summarizes the main results of our evolutionary calculations. Lifetimes listed include: τ_H, the core hydrogen-burning lifetime; τ_{He}, the core helium-burning lifetime; and τ_b and τ_r, the fractions of the core helium-burning lifetime that are spent in the blue supergiant and red supergiant configurations, respectively. Phases of evolution after core helium burning are so drastically shortened by neutrino emission that we have ignored them here. The range of effective temperatures occupied in the blue supergiant configuration during the slow stages of core helium burning is also tabulated.

As already noted, the evolutionary sequences display almost no sensitivity to the uncertainties associated with the initial iron abundance, molecular opacities, convective mixing length, and stellar wind mass-loss rate, except in their roles of suppressing or triggering a blue loop. Although an increase of Z_e from 0.02 to 0.03 reduces the effective temperatures of blue supergiants depleting core helium and also lowers the ratio of lifetimes in the blue and red regions during core helium burning, these reductions are too small to detect observationally. A switch of the $^{12}C(\alpha, \gamma)^{16}O$ reaction rate from "large" to "small" produces effects that are even more inconspicuous (Table 2). Bressan et al. (1993) and El Eid (1994) have computed evolutionary sequences using, in effect, the equivalent of the Ledoux criterion (because semiconvective mixing was not allowed); agreement with our analogous sequences is excellent. Likewise, the sequences based on the Schwarzschild criterion by Mowlavi & Forestini (1994) closely resemble ours.

The only major source of possible uncertainty seems to be the extent of convective core overshooting. Inclusion of overshooting lessens convective instability in the layers with a μ gradient (Cloutman & Whitaker 1980) and reddens models in the post–main-sequence stages (Matraka, Wassermann, & Weigert 1982). Our overshooting sequences based on the Ledoux criterion (Table 2) follow this expected pattern, as do the analogous sequences calculated with somewhat more overshooting by Bressan et al. (1993). The overshooting sequences calculated by Schaller et al. (1992), Meynet et al. (1994), and Li & Gong (1994), using the Schwarzschild criterion, also become redder, but the amount of reddening that they derived may be too large, because they apparently did not allow for upward and downward overshooting from the FCZ. Since it is unclear whether the same value of d/H_P ought to be used at the upper

TABLE 1

SUMMARY OF THE EVOLUTIONARY SEQUENCES OF STELLAR MODELS

Initial $M/M_\odot$	Z_e	Criterion	τ_H (10^6 yr)	τ_{He}/τ_H	τ_b/τ_r	Blue Tip log T_e	Blue Tip log $(L/L_\odot)$	Blue/Yellow Edge log T_e	Blue/Yellow Edge log $(L/L_\odot)$
10............	0.02	S	19.89	0.141	3.22	4.16	4.21	3.95	4.19
	0.03	S	21.13	0.145	1.96	4.11	4.14	3.83	4.06
	0.02	L	19.89	0.135	0.70	4.13	4.23	3.90	4.15
	0.03	L	21.13	0.139	1.01	4.09	4.15	3.79	4.04
15............	0.02	S	10.90	0.120	0.35	4.03	4.74	3.69	4.73
	0.03	S	11.28	0.128	0.12	3.96	4.68	3.67	4.65
	0.02	L	10.90	0.105	0.62	4.20	4.73	4.00	4.69
	0.03	L	11.28	0.105	0.61	4.17	4.68	3.93	4.63
20............	0.02	S	8.08	0.109	2.59	4.12	5.04	3.70	5.06
	0.03	S	8.12	0.102	1.92	4.05	5.01	3.68	5.01
	0.02	L	7.76	0.099	0.92	4.24	5.06	4.10	5.04
	0.03	L	7.98	0.097	0.43	4.20	5.03	4.05	5.00

NOTE.—Dividing line between "blue" and "red" is taken at log T_e = 3.65.

TABLE 2

SUPPLEMENTARY EVOLUTIONARY SEQUENCES OF STELLAR MODELS WITH $Z_e = 0.02$

Set	Initial $M/M_\odot$	Criterion	τ_H (10^6 yr)	τ_{He}/τ_H	τ_b/τ_r	Blue Tip log T_e	Blue Tip log $(L/L_\odot)$	Blue/Yellow Edge log T_e	Blue/Yellow Edge log $(L/L_\odot)$
1..........	10	L	19.89	0.126	0.38	4.09	4.22	3.92	4.15
	15	S	10.90	0.114	0.34	4.03	4.73	3.68	4.71
	15	L	10.90	0.098	0.71	4.20	4.73	4.02	4.70
	20	S	8.08	0.096	4.34	4.12	5.04	3.69	5.05
2..........	10	L	22.79	0.089	0.63	4.04	4.33	3.86	4.31
	15	L	12.25	0.078	0.69	4.10	4.84	3.87	4.81
	20	L	8.50	0.078	0.00	...	...	...	...

NOTE.—Set 1 is based on the "small" $^{12}C(\alpha, \gamma)^{16}O$ rate. Set 2 is based on convective core overshooting with $d/H_P = 0.2$. Other physical input parameters are the same as those used for Table 1.

boundary of the FCZ, the lower boundary of the FCZ, and the boundary of the convective core, we have not attempted to refine their results. However, by adopting the same value at all convective boundaries, Mowlavi & Forestini (1994) have obtained less reddening.

4. COMPARISON WITH CLUSTER H-R DIAGRAMS

Young star clusters in the Galaxy are not as rich in evolved supergiants as is NGC 330 in the SMC. In the case of the Galaxy, a meaningful H-R diagram must therefore be prepared by pooling the contents of several clusters, since the richer associations present too many problems to be really useful, with the exception of one well-observed feature that will be discussed below (§ 5.1). The basic observational data possessing acceptably high quality have been collected for six Galactic clusters in Table 6 of Stothers (1991). These data include transformed coordinates for 14 evolved cluster supergiants, which are plotted here as circles in Figures 2–5. The estimated mean errors are ± 0.1 in log $(L/L_\odot)$ and ± 0.03 in log T_e. None of these stars is known to possess a close binary companion, and so their evolution is expected to have proceeded normally.

First of all, we note in Figures 2–5 that the theoretical evolutionary tracks for stars of 10 $M_\odot$, which are relatively insensitive to the choice of the criterion for convection, satisfy the observations of the three supergiants with the lowest luminosities. The predicted blue-to-red number ratio (roughly 1:1, within a factor of 2) and the predicted effective temperatures of the blue and red supergiants are confirmed. Despite the very small number of stars, this agreement lends plausibility to the tests for the criterion for convection that will be made at higher luminosities, specifically for the 11 stars with log $(L/L_\odot) > 4.3$.

As a group, the new theoretical models for blue supergiants in the mass range 15–20 $M_\odot$ are markedly cooler than observations indicate if the Schwarzschild criterion for convection is adopted. Although the predicted blue-to-red number ratio is acceptably large (2:1) at 20 $M_\odot$, it falls far too short (0.2:1) at 15 $M_\odot$. Observations in this mass range indicate a ratio of about 1:1.

On the other hand, the Ledoux criterion correctly predicts not only the effective temperature range observed for blue supergiants but also a reasonable approximation (0.6:1) of the observed blue-to-red number ratio, which is a very difficult quantity to predict accurately (Chin & Stothers 1991). Thus, we are able to confirm our previous conclusion about the correctness, in effect, of the Ledoux criterion.

The luminosity difference between blue and red supergiants of the same initial mass provides another potential discriminant of the criterion for convection. For the Ledoux criterion, the mean luminosity of the blue supergiants should be brighter by 0.1 dex than the mean luminosity of the red ones, whereas a near-equality of the luminosities should prevail in the case of the Schwarzschild criterion. Taking separate averages over observed blue and red supergiants, we find that the 11 plotted supergiants with log $(L/L_\odot) > 4.3$ exhibit an average luminosity difference of $\Delta\langle \log (L/L_\odot)\rangle = 0.3 \pm 0.2$. The mean error due to the scatter of the observed luminosities, however, turns out to be too large to establish a meaningful difference.

In making these tests, we have used stellar models that did not include convective core overshooting. With moderate core overshooting included, post–main sequence models become redder on the H-R diagram (§ 3). A shift in this general direction, however, would exacerbate the disagreement with observations already obtained for the Schwarzschild criterion and would degrade the agreement found for the Ledoux criterion. Overshooting is therefore probably rather small, as has already been indicated by independent evidence (§ 1).

5. OTHER TESTS FOR THE CRITERION FOR CONVECTION

5.1. *Association H-R Diagrams*

The H-R diagrams of Galactic stellar associations have long been used to study the question of the criterion for convection. Especially popular has been the composite H-R diagram containing supergiant members of many associations. The chief problem with this approach has been the inability to discern in association data an expected gap that should, theoretically, separate blue supergiants that are still burning core hydrogen from those evolving in more advanced phases (Stothers 1972a, Fig. 4). Irreducible photometric and spectroscopic scatter of the stars due to measurement uncertainties, as well as the likely presence of some abnormally evolved stars belonging to unrecognized interacting close binary systems (e.g., Tuchman & Wheeler 1990; Rathnasree & Ray 1992; Langer 1992), affects the generally less well-observed association data much more than the highly refined cluster data. These difficulties seriously jeopardize any observational estimation of both the blue edge of the core–helium-burning region on the H-R diagram and the blue-to-red number ratio of evolved supergiants. Thus, Humphreys (1970, 1978) and Humphreys & McElroy (1984) derived a blue-to-red number ratio of between 7:1 and 26:1 for association supergiants of ~15–20 $M_\odot$, whereas our use of only the most refined, though much less abundant, cluster data (§ 4) indicates a ratio of 1:1 for truly evolved stars in this mass range. The confusion introduced by the association data for this ratio has been a partial factor in inducing many authors to tilt toward or even to prefer the Schwarzschild criterion (Simpson 1971; Ziólkowski 1972; Robertson 1973; Barbaro et al. 1973; Lamb, Iben, & Howard 1976; Chiosi, Nasi, & Sreenivasan 1978; Maeder 1981; Brunish & Truran 1982).

A better diagnostic feature in the case of the stellar associations is the observed Hertzprung gap separating the blue supergiants from the red supergiants. The red edge of the gap is nearly vertical at log $T_e \approx 3.55$, while its blue edge runs up vertically along log $T_e \approx 3.95$ to log $(L/L_\odot) \approx 4.8$, whereupon it veers off to the upper left (Blaha & Humphreys 1989). Long ago, we noted this sloping trend of the brightest blue supergiants from Humphreys's (1970) original composite H-R diagram (Stothers & Chin 1975, 1976), while a very similar feature among the blue supergiants in the LMC was noted by Hutchings (1976) and Fitzpatrick & Garmany (1990), who called the apparent yellow edge of the blue distribution a "ledge."

Our original stellar models based on the Ledoux criterion predicted an approximately correct slope of this ledge, but placed the whole feature at much too high an effective temperature (Stothers & Chin 1975, 1976). Subsequent models with better opacities have significantly lowered the expected effective temperature (Weiss 1989; Langer 1991a, b; Arnett 1991), although Tuchman & Wheeler (1989a, b, 1990) still prefer the Schwarzschild criterion, in part because it yields a very cool ledge even if it predicts an incorrect slope of the ledge. By using the most recent opacities and mass-loss rates, our new stellar models now come down strongly in favor of the Ledoux criterion. This can be easily seen in Figures 2–5, where the dashed line represents the observed ledge. Mowlavi & Forestini (1994) have reached a somewhat similar conclusion (their "semiconvective" case).

An attempt was made some years ago to determine the mean luminosity difference between blue and red supergiants of approximately the same initial mass in stellar associations (Stothers & Lloyd Evans 1970). Although the Ledoux criterion was supported, too many resolution problems remain to give this result any significant weight.

5.2. *Fossil Dust Shells*

Yellow supergiants of luminosity class Ia (initial masses greater than 15 $M_\odot$) show, in many instances, strong infrared excesses, indicating the presence of a dusty circumstellar shell. It is very difficult to understand how such a shell could have formed in the hot environment of a yellow supergiant. If, however, these stars had recently been red supergiants, which readily create dust shells through their prodigious mass loss by stellar wind, the infrared excess could be explained (Salpeter, in Ney 1972). Examined quantitatively, the observed frequency of infrared excesses around F, G, and K supergiants of luminosity class Ia strongly supports the idea of a prior M-supergiant phase, and hence favors the Ledoux criterion for convection (Stothers 1975; see also Mutel et al. 1979; Odenwald 1986; Stencel, Pesce, & Bauer 1989; Sowell 1990; Roche, Aitken, & Smith 1993).

5.3. *Secular Changes*

Historical observations of a number of very luminous supergiants have demonstrated the existence of slow, possibly secular, changes that might indicate rapid evolutionary development. In those cases where a stellar expansion is observed, only a knowledge of the precise rate of radius increase could possibly discriminate between the two criteria for convection. A contraction, on the other hand, would presumably point to the Ledoux criterion. To perform such a test for the criterion for convection, initial stellar masses must be large enough (>12 $M_\odot$) to be able to discriminate a difference, but small enough that mass-loss effects do not preclude the possible formation of a large FCZ (Chiosi & Nasi 1974).

The potentially most interesting observational data turn out to be disappointingly ambiguous about the direction and rate of evolution. Several decades-long time series of observations have been unable to resolve a secular trend in even the best cases—the cases for α Cyg (A2 Ia) (Liller & Liller 1965), ρ Cas (F8 Ia–0) (Beardsley 1961; Zsoldos & Percy 1991), and HR 8752 (G4 Ia–0) (Zsoldos 1986; Percy & Zsoldos 1992). A much longer baseline, using observed modern magnitudes and magnitudes from Ptolemy's *Almagest*, has enable Mayer (1984) to derive a statistically valid result for luminosity class Ia supergiants with spectral types B0 to F8: a mean visual brightening of 0.037 ± 0.007 mag per century. Since evolution proceeds at nearly constant luminosity for such massive stars, this brightening would have been caused mainly by changes of bolometric correction, implying a drop of effective temperature and hence an expansion. However, this numerical result is too general to be useful for our present purposes.

A more specific result, 0.15 ± 0.02 mag per century, was recently derived for P Cyg (B1 Ia+) by de Groot & Lamers (1992) and Lamers & de Groot (1992), using three centuries' accumulation of photometric data. From their result, El Eid & Hartmann (1993) deduced provisional evidence for the Schwarzschild criterion by assuming that P Cyg is just leaving the main sequence and by noting that the Ledoux criterion would predict a rate of expansion smaller by an order of magnitude. P Cyg, however, is probably a considerably more evolved object than El Eid & Hartmann assumed. As a classical luminous blue variable, it is known to have suffered two massive outbursts during the last three centuries (Lamers & de Groot 1992), and it presently exhibits a noticeable overabundance of nitrogen in its photosphere (Luud 1967) as well as in its ejected nebula (Johnson et al. 1992). Therefore, the present mass of the star is probably much smaller than the main-sequence value of ~ 50 $M_\odot$ assumed by El Eid & Hartmann, with the consequence that its thin remnant envelope should be able to expand rather rapidly regardless of the criterion for convection (Stothers & Chin 1994). The atmospherically derived mass of the star is in fact 23 ± 2 $M_\odot$ (Pauldrach & Puls 1990).

Very massive stars that are truly just evolving off the main sequence present a second potential test of the criterion for convection, although it is at present impractical to perform it. If a large FCZ has developed, the hydrogen-burning shell should be thermally pulsing. The observable surface luminosity amplitude, though, would be at most a few hundredths of a magnitude, spanning a period of several centuries (Stothers & Chin 1972, 1983).

A more practicable test for the direction of evolution can be based on the observed secular period changes in very luminous classical Cepheid variables (Ziólkowski 1972). A lengthening pulsation period would indicate expansion, while a shortening period would point to contraction. The most luminous classical Cepheid with adequate period measurements is the 45 day variable SV Vul, which shows a secular period decrease (Szabados 1983; Saitou 1989). The inferred mass of this star, however, lies in the range 10–13 $M_\odot$ (Fernie 1979), which is somewhat too small a mass for us to conclude with assurance that the Ledoux criterion is implied. A period increase, like that of the 68 day variable S Vul (Berdnikov 1994), cannot discriminate the criterion. Longer period classical Cepheids should be critically examined for secular period changes, in spite of the well-known difficulties posed by the irregularities of their observed pulsation periods.

5.4. *Binary Frequencies*

The expansion of a star's envelope in a close binary system is ultimately limited by the size of the star's Roche lobe. Consequently, in a random sample of binary systems compared to single stars, the red supergiant configuration should be observed relatively infrequently, and no blue supergiant evolving on a blue loop could show a short orbital period. Depending on whether the Ledoux criterion or the Schwarzschild criterion is adopted, this physical constraint leads to differing predictions for the observable percentages of close binary systems occurring among luminous supergiants of various spectral types. Although preliminary observational data bearing on this question have pointed to the correctness of the Ledoux criterion (Stothers 1969; Stothers & Lloyd Evans 1970), the problem of removing statistical bias due to incompleteness and the difficulty of identifying among the blue supergiants those objects that are actual remnants of mass exchange have somewhat weakened this test (Chiosi & Summa 1970; Lloyd Evans 1971; Stothers 1972b; Burki & Mayor 1983; Vanbeveren 1988).

5.5. *Surface Helium Abundance after Binary Mass Exchange*

If a massive star evolving in a close binary system experiences Roche lobe overflow after the end of central hydrogen burning, its outer envelope will be stripped off, exposing deep layers that thinly blanket the hydrogen-burning shell. Depending on the presence or absence of an FCZ, the new surface helium abundance should be slightly higher than normal (for the Schwarzschild criterion) or very high (for the Ledoux criterion) (cf. Fig. 1). In the only suitable observed object, β Lyr, the surface helium abundance of the primary turns out to be very high, $Y \approx 0.8$–0.9 (Boyarchuk 1960; Hack & Job 1965). This observation would seem to favor the Ledoux criterion (Stothers & Chin 1976). Unfortunately, the original mass of the primary probably lay in the range 9–13 $M_\odot$ (Stothers 1972c; Kříž 1974; Wilson 1974; Ziółkowski 1976; Skul'skii 1992), which is too low a value to enable our drawing a firm conclusion about the criterion for convection. Massive Wolf-Rayet stars in close binary systems are potentially better candidates, but their observed surface helium enrichment may have resulted from stellar wind mass loss after the binary mass exchange.

The observed effective temperature of a stripped primary can also indicate the criterion for convection (Massevitch & Tutukov 1974). Unless, however, the mass exchange is known to have occurred very recently, continuing evolution of the primary will alter the star's effective temperature, thereby invalidating the test in actual practice.

5.6. *Surface Nitrogen Abundances*

Supergiants of high mass that are evolving along blue loops on the H-R diagram are expected to show at their surfaces the chemical effects of deep convective envelope mixing during the prior red supergiant phase. The most easily measurable chemical anomaly would be a strong enhancement of nitrogen relative to carbon and oxygen, caused by convective dredge-up of the products of former core hydrogen burning. On the other hand, blue supergiants that have not yet experienced a red phase should exhibit normal nitrogen abundances (Wallerstein, in Stothers & Lloyd Evans 1970; Simpson 1971). Some evidence favoring the Ledoux criterion has been found from the relative nitrogen overabundances seen in a number of luminous blue supergiants (Stothers & Lloyd Evans 1970; Walborn 1976; Fitzpatrick & Garmany 1990; Langer 1991a).

Many questions remain, however: Are these particular supergiants massive enough to use in a test for the criterion for convection? Are they so massive that heavy stellar wind mass outflow has exposed their interior hydrogen-processed layers? Have they suffered mass loss, or perhaps mass accretion, in a close binary mass exchange? Has rotational mixing, tidally induced mixing, or radiative diffusion brought the nitrogen-enriched material up from the stellar core? Was the star's natal material itself nitrogen-rich? How accurate are the observational determinations of the nitrogen abundances? These questions must all be answered individually for the blue supergiants being studied before the suggested test for the criterion for convection can be safely made.

5.7. *SN 1987A in the LMC*

Supernova 1987A in the LMC had as an immediate precursor a B3 supergiant, Sk $-69°202$ (see the review by Arnett et al. 1989). The supernova's light curve implies a substantial mass of at least ~ 5 $M_\odot$ still contained in the precursor's envelope at the time of explosion. This large value is supported by the rather normal-looking spectroscopic appearance of Sk $-69°202$, whose initial mass, estimated from its luminosity, must have been ~ 20 $M_\odot$. Normal-appearing nitrogen lines in the blue supergiant, however, could imply an actual overabundance of this element (Walborn 1976; Langer 1991a). A red supergiant phase preceding the final blue supergiant phase is inferred from observations of a distant nebula surrounding the exploded star. Light echoes reveal the presence of the nebula, which exhibits overabundances of nitrogen, helium, and the *s*-process element barium. In addition, the LMC is known to contain both blue and red supergiants of initially ~ 20 $M_\odot$.

Various models have been proposed to explain these observations. In most of the published studies, a red supergiant phase in a star of initially ~ 20 $M_\odot$ with LMC metallicity is taken to imply the correctness, in effect, of the Ledoux criterion for convection (Woosley, Pinto, & Weaver 1988; Woosley, Pinto, & Ensman 1988; Woosley 1988; Weiss 1989; Langer, El Eid, & Baraffe 1989; Langer 1991a, b; Arnett 1987, 1991). The reason is that most of the standard evolutionary tracks based on the Schwarzschild criterion remain blue right up to the time of explosion (Hillebrandt et al. 1987; Truran & Weiss 1987; Weiss, Hillebrandt, & Truran 1988; Saio, Kato, & Nomoto 1988; Saio, Nomoto, & Kato 1988; Weiss 1989; see also Barkat & Wheeler 1988, 1989a, b; Tuchman & Wheeler 1989a, b, 1990).

This conclusion, however, is not required. To account for the red supergiant phase using the Schwarzschild criterion, Maeder (1987), Wood & Faulkner (1987), and Saio et al. (1988a, b) proposed substantial mass loss during the preceding blue phase, which forces the star to become red, before it finally ends up blue again. Moderate convective core overshooting has also been assumed in obtaining a red supergiant phase (Schaerer et al. 1993). Another consideration is that the red supergiant phase itself is only indirectly inferred for SN 1987A, and may never have occurred if the blue star's envelope expansion had been inhibited by the presence of a close binary companion. If this was the case and Roche lobe overflow did take place, many of the observed anomalies can be naturally explained, including the distant nebula, the large nitrogen overabundance, the substantial final envelope mass, and the glowing ring of material around the exploded supernova, which is otherwise difficult to explain (see the review in Podsiadlowski 1992). Although the occurrence of a prior phase of mass exchange does not obviously indicate which criterion for convection is correct, Staritsin & Tutukov (1989) favored the Schwarzschild criterion because it yields a not-too-small final hydrogen abundance at the stellar surface.

In retrospect, it is perhaps too much to expect that the examination of one star, in a terminal stage of evolution, could have led to a decisive test of the criterion for convection. SN 1987A is too complex for this purpose. In fact, most of the published arguments that were based nominally on SN 1987A have relied more on an analogy between Sk $-69°202$ and the other observed LMC supergiants of $\sim 20\ M_\odot$ (cf. § 5.1).

6. CONCLUSION

Ten different tests for the effective criterion for convection and semiconvection in the inhomogeneous layers of an evolved star have been considered in this paper. Three of these tests were applied twice: first to Galactic cluster members and then to Galactic association members. One test made use of SN 1987A in the LMC. Although abundant new material has been presented, much work by many other authors was incorporated in this investigation.

Results of all 13 tests are summarized in Table 3. At least three, and possibly as many as five, tests support the Ledoux criterion. Although the eight remaining tests show inconclusive outcomes, no test result definitely supports the Schwarzschild criterion (insofar as its use leads to the formation of a large FCZ). We therefore confirm our original conjecture about the appropriateness of the Ledoux criterion based on the observed presence of many red supergiants in the h and χ Per association (Stothers & Chin 1968). These results agree also with more recent work by ourselves, Arnett, Fitzpatrick, Garmany, Langer, and Weiss. Even though further work might be able to force a decision in some or all of the indeterminate cases in Table 3, no change of basic conclusion is anticipated.

TABLE 3

RESULTS OF TESTS FOR THE CRITERION FOR CONVECTION IN MASSIVE SUPERGIANTS

Test	Criterion
Cluster H-R diagrams (Galaxy):	
Effective temperature range of blue supergiants	L
Relative luminosities of blue and red supergiants	Indeterminate
Relative numbers of blue and red supergiants	L?
Association H-R diagrams (Galaxy):	
Effective temperature range of blue supergiants	L
Relative luminosities of blue and red supergiants	Indeterminate
Relative numbers of blue and red supergiants	Indeterminate
Fossil dust shells	L
Secular changes:	
Photometric and spectroscopic changes	Indeterminate
Period changes in classical Cepheids	Indeterminate
Binary frequencies	L?
Surface helium abundance after binary mass exchange	Indeterminate
Surface nitrogen abundance of blue supergiants	Indeterminate
SN 1987A in the Large Magellanic Cloud	Indeterminate

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